VIDYAPEETH ACADEMY
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# PAPER-1(B.E./B. TECH.)

# JEE (Main) 2021

## Questions & Solutions

(Reproduced from memory retention)

Date : 24 February, 2021 (SHIFT-2) Time ; (3.00 am to 6.00 pm)

Duration : 3 Hours | Max. Marks : 300

## SUBJECT : MATHEMATICS

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# MATHEMATICS

**1.** Find the value of ${}^{n+1}C_2 + 2\,({}^{2}C_2 + {}^{3}C_2 + \ldots\ldots + {}^{n}C_2) = ?$

(1) $\dfrac{n(n+1)(2n-1)}{6}$ (2) $\dfrac{n(n+1)(2n+1)}{6}$

(3) $\dfrac{(n-1)n(n+1)}{6}$ (4) $\dfrac{n(n+1)}{2}$

**Ans.** (2)

**Sol.** $S = {}^{2}C_2 + {}^{3}C_2 + \ldots\ldots + {}^{n}C_2 = {}^{n+1}C_3$

$\therefore\ {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3 = {}^{n+2}C_3 + {}^{n+1}C_3$

$= \dfrac{(n+1)!}{3!(n-1)!} + \dfrac{(n+1)!}{3!(n-2)!}$

$= \dfrac{(n+2)(n+1)n}{6} + \dfrac{(n+1)(n)(n-1)}{6} = \dfrac{n(n+1)}{6}\,(2n+1)$

**2.** If A and B are subset s of X = {1,2,3,4,5} then find the probability such that $n(A \cap B) = 2$.

(1) $\dfrac{65}{2^7}$ (2) $\dfrac{65}{2^9}$ (3) $\dfrac{35}{2^9}$ (4) $\dfrac{135}{2^9}$

**Ans.** (4)

**Sol.** Required probability

$= \dfrac{{}^{5}C_2 \times 3^3}{4^5}$

$= \dfrac{10 \times 27}{2^{10}} = \dfrac{135}{2^9}$

**3.** Given $f(0) = 1$, $f(2) = e^2$ also $f'(x) = f'(2-x)$, then the value of $\int_0^2 f(x)dx$ is

(1) $1 - e^2$ (2) $1 + e^2$ (3) $e$ (4) $e^2$

**Ans.** (2)

**Sol.** $f'(x) = f'(2-x)$

On integrating both side $f(x) = -f(2-x) + c$

put $x = 0$

$f(0) + f(2) = c \quad \Rightarrow \quad c = 1 + e^2$

$\Rightarrow\ f(x) + f(2-x) = 1 + e^2 \ldots\ldots(i)$

$I = \int_0^2 f(x)\,dx = \int_0^1 \{f(x) + f(2-x)\}\,dx = (1 + e^2)$

**4.** A curve y = f(x) passing through the point (1,2) satisfies the differential equation $x\frac{dy}{dx}+y=bx^4$ such that $\int_1^2 f(y)dy=\frac{62}{5}$. The value of b is

(1) 10 (2) 11 (3) $\frac{32}{5}$ (4) $\frac{62}{5}$

**Ans.** (1)

**Sol.** $\frac{dy}{dx}+\frac{y}{x}=6x^3$

I.F. = $e^{\int\frac{dx}{x}}=x$

$\therefore$ yx = $\int bx^4dx=\frac{bx^5}{5}+C$

Passes through (1,2) , we get

$2=\frac{b}{5}+C$ ..(i)

Also , $\int_1^2\left(\frac{bx^4}{5}+\frac{C}{x}\right)dx=\frac{62}{5}$

$\Rightarrow \frac{b}{25}\times 32+C\ell n2-\frac{b}{25}=\frac{62}{5}$

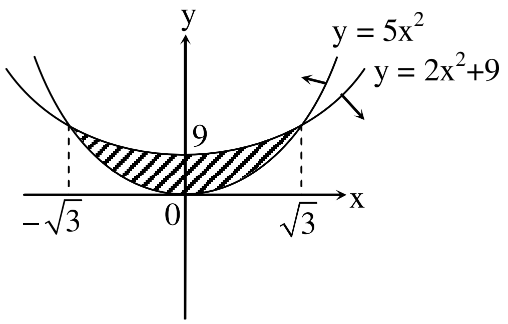
$\Rightarrow C=0$ & $b=10$

**5.** The area of the region defined by $5x^2 \le y \le 2x^2+9$ is

(1) $6\sqrt{3}$ (2) $12\sqrt{3}$ (3) $18\sqrt{3}$ (4) $9\sqrt{3}$

**Ans.** (2)

**Sol.**




Required area

$= 2\int_0^{\sqrt{3}}(2x^2+9-5x^2)dx$

$= 2\int_0^{\sqrt{3}}(9-3x^2)dx$

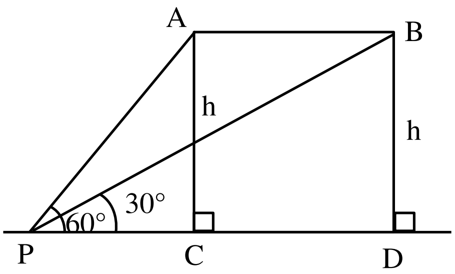
$=2|9x-x^3|_0^{\sqrt{3}} = 12\sqrt{3}$

**6.** A aeroplane is flying horizontally with speed of 432 km/hr at height h meter from ground. Its angle of elevation from a point on ground is 60°. After 20 sec its angle of elevation from same point is 30° then the height 'h' is equal to

(1) $1200\sqrt{3}$ (2) $600\sqrt{3}$ (3) $1800\sqrt{3}$ (4) $1000\sqrt{3}$

**Ans.** (1)

**Sol.**




$v = 432 \times \frac{1000}{60 \times 60}$ m/sec = 120 m/sec

Distance AB = v × 20 = 2400 meter

In $\Delta PAC$

$\tan 60° = \frac{h}{PC} \Rightarrow PC = \frac{h}{\sqrt{3}}$

In $\Delta PBD$

$\tan 30° = \frac{h}{PD} \Rightarrow PD = \sqrt{3}h$

PD = PC + CD

$\sqrt{3}h = \frac{h}{\sqrt{3}} + 2400 \Rightarrow \frac{2h}{\sqrt{3}} = 2400$

$h = 1200\sqrt{3}$ meter

**7.** A curve $y = ax^2 + bx + c$ passing through the point (1, 2) has slope at origin equal to 1. then ordered triplet (a, b, c) may be

(1) (1, 1, 0) (2) $\left(\frac{1}{2}, 1, 0\right)$ (3) $\left(-\frac{1}{2}, 1, 1\right)$ (4) (2, –1, 0)

**Ans.** (1)

**Sol.** $2 = a + b + c$ ........ (i)

$\frac{dy}{dx} = 2ax + b \Rightarrow \left.\frac{dy}{dx}\right|_{(0,0)} = 1$

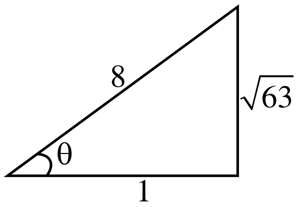
$\Rightarrow b = 1 \Rightarrow a + c = 1$

**8.** The value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is

(1) $\frac{1}{\sqrt{7}}$ (2) $\frac{1}{\sqrt{5}}$ (3) $\frac{2}{\sqrt{3}}$ (4) none of these

**Ans.** (1)

**Sol.** Let $\sin^{-1}\frac{\sqrt{63}}{8} = \theta \Rightarrow \sin\theta = \frac{\sqrt{63}}{8}$




$$\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right) = \tan\left(\frac{\theta}{4}\right) = \frac{1-\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = \frac{1-\sqrt{\frac{1+\cos\theta}{2}}}{\sqrt{\frac{1-\cos\theta}{2}}} = \frac{1-\frac{3}{4}}{\frac{\sqrt{7}}{4}} = \frac{1}{\sqrt{7}}$$

**9.** The value of $\int_1^3 [x^2-2x-2]dx$ ([.] denotes greatest integers function)

(1) – 4 (2) –5 (3) $-1-\sqrt{2}-\sqrt{3}$ (4) $1-\sqrt{2}-\sqrt{3}$

**Ans.** (3)

**Sol.** $I = \int_1^3 -3dx + \int_1^3 [(x-1)^2]\,dx$ $\quad x - 1 = t;\ dx = dt$

$I = (-6) + \int_0^2 [t^2]dt$

$I = -6 + \int_0^1 0\,dt + \int_1^{\sqrt{2}} 1\,dt + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dt + \int_{\sqrt{3}}^{2} 3\,dt$

$I = -6 + \left(\sqrt{2}-1\right) + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$

$I = -1 - \sqrt{2} - \sqrt{3}$

**10.** Which of the following conic has tangent '$x+\sqrt{3}y-2\sqrt{3}$' at point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?

(1) $x^2 + 9y^2 = 9$ (2) $y^2 = \frac{x}{6\sqrt{3}}$ (3) $x^2 - 9y^2 = 10$ (4) $x^2 = \frac{y}{6\sqrt{3}}$

**Ans.** (1)

**Sol.** tangent to $x^2 + 9y^2 = a$ at point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$ is $x\left(\frac{3\sqrt{3}}{2}\right) + 9y\left(\frac{1}{2}\right) = 9$

$\Rightarrow$ option (1) is true

**11.** The negation of the statement $\sim p \wedge (p \vee q)$ is

(1) $p \wedge \sim q$ (2) $p \vee \sim q$ (3) $\sim p \wedge q$ (4) $\sim p \vee \sim q$

**Ans.** (2)

**Sol.** $\sim(\sim p \wedge (p \vee q))$

$= \sim((\sim p \wedge p) \vee (\sim p \wedge q))$

$= \sim(\sim p \wedge q) = p \vee \sim q$

**12.** Equation of plane passing through (1, 0, 2) and line of intersection of planes $\vec{r}\,.(\hat{i}+\hat{j}+\hat{k}) = 1$ and $\vec{r}\,.(\hat{i}-2\hat{j}) = -2$ is

(1) $\vec{r}\,.(\hat{i}+7\hat{j}+3\hat{k}) = 7$ (2) $\vec{r}\,.(3\hat{i}+10\hat{j}+3\hat{k}) = 7$

(3) $\vec{r}\,.(\hat{i}+\hat{j}-3\hat{k}) = 4$ (4) $\vec{r}\,.(\hat{i}+4\hat{j}-\hat{k}) = -7$

**Ans.** (1)

**Sol.** Plane passing through intersection of plane is

$\{\vec{r}\,.(\hat{i}+\hat{j}+\hat{k}) = -1\} + \lambda\{\vec{r}\,.(\hat{i}-2\hat{j})+2\} = 0$

Passes through $\hat{i}+2\hat{k}$, we get

$(3 - 1) + \lambda\,(\lambda + 2) = 0 \qquad \Rightarrow \qquad \lambda = -\dfrac{2}{3}$

Hence, equation of plane is $\quad 3\{\vec{r}\,.(\hat{i}+\hat{j}+\hat{k})-1\} - 2\{\vec{r}\,.(\hat{i}-2\hat{j})+2\} = 0$

$\Rightarrow \quad \vec{r}\,.(\hat{i}+7\hat{j}+3\hat{k}) = 7$

**13.** A is 3 × 3 square matrix and B is 3 × 3 skew symmetric matrix and X is a 3 × 1 matrix, then equation $(A^2B^2 - B^2A^2)\,X = 0$ (Where O is a null matrix) has/have

(1) Infinite solution (2) No Solution

(3) Exactly one solution (4) Exactly two solution

**Ans.** (1)

**Sol.** $A^T = A,\ B^T = -B$

Let $A^2B^2 - B^2A^2 = P$

$P^T = (A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T$

$= (B^2)^T (A^2)^T - (A^2)^T (B^2)^T$

$= B^2A^2 - A^2B^2$

$\Rightarrow$ P is skew-symmetric matrix

$\Rightarrow$ $|P| = 0$

Hence PX = 0 have infinite solution

**14.** If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, f(0) = 1 and f'(0) = 2 then f(1) belongs to interval

(1) [6, 9] (2) [9, 12] (3) [8, 10] (4) [5, 7]

**Ans.** (1)

**Sol.** Given $f(x)\, f''(x) - (f'(x))^2 = 0$

Let $h(x) = \dfrac{f(x)}{f'(x)}$

$\Rightarrow h'(x) = 0$ $\Rightarrow h(x) = k$

$\Rightarrow \dfrac{f(x)}{f'(x)} = k$ $\Rightarrow f'(x) = k\, f'(x)$

$\Rightarrow f(x) = k\, f'(0)$ $\Rightarrow 1 = k(2) \Rightarrow k = \dfrac{1}{2}$

New $f(x) = \dfrac{1}{2} f'(x) \Rightarrow \int 2dx = \int \dfrac{f'(x)}{f(x)} dx$

$\Rightarrow 2x = \ln|f(x)| + C$

As $f(0) = 1 \Rightarrow C = 0$

$\Rightarrow 2x = \ln|f(x)| \Rightarrow f(x) = \pm e^{2x}$

As $f(0) = 1 \Rightarrow f(x) = e^{2x} \Rightarrow f(1) = e^2$

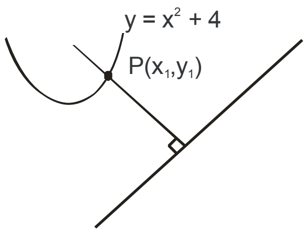
**15.** Find a point on the curve $y = x^2 + 4$ which is at shortest distance from the line $y = 4x - 1$.

(1) (2,8) (2) (1,5) (3) (3,13) (4) (–1,5)

**Ans.** (1)

**Sol.** $\left.\dfrac{dy}{dx}\right|_p = 4$

$\therefore 2x_1 = 4$




$\Rightarrow x_1 = 2$

$\therefore$ Point will be (2,8)

**16.** Let $f(x) = \begin{cases} -55x & ; \quad x < -5 \\ 2x^3 - 3x^2 - 120x & ; \quad -5 \le x < 4 \\ 2x^3 - 3x^2 - 36x + 10 & ; \quad x \ge 4 \end{cases}$.

Then interval in which f(x) is monotonically increasing is

(1) $(-5, -4) \cup (4, \infty)$ (2) $(-\infty, -4) \cup (5, \infty)$

(3) $(-5, 4) \cup (5, \infty)$ (4) $(-5, -4) \cup (3, \infty)$

**Ans.** (1)

**Sol.** $f'(x) = \begin{cases} -55 & ; \quad x < -5 \\ 6(x^2 - x - 20) & ; \quad -5 < x < 4 \\ 6(x^2 - x - 6) & ; \quad x > 4 \end{cases}$

$f'(x) = \begin{cases} -55 & ; \quad x < -5 \\ 6(x-5)(x+4) & ; \quad -5 < x < 4 \\ 6(x-3)(x+2) & ; \quad x > 4 \end{cases}$

Hence, f(x) is monotonically increasing is $(-5, -4) \cup (4, \infty)$

**17.** If a, b, c are in A.P. & centroid of the triangle with vertices (a, c), (a, b), (2, b) is $\left(\frac{10}{3}, \frac{7}{3}\right)$ and $\alpha, \beta$ are roots of the equation $ax^2 + bx + 1 = 0$, then $\alpha^2 + \beta^2 - \alpha\beta$ equals

(1) $-\frac{71}{256}$ (2) $\frac{71}{256}$ (3) $\frac{69}{256}$ (4) $-\frac{69}{256}$

**Ans.** (1)

**Sol.** $2b = a + c$

$\frac{2a+2}{3} = \frac{10}{3}$ and $\frac{2b+c}{3} = \frac{7}{3}$

$\Rightarrow \quad a = 4 \qquad \left.\begin{matrix} 2b + c = 7 \\ 2b - c = 4 \end{matrix}\right\}$ solving,

$b = \frac{11}{4} \qquad c = \frac{3}{2}$

$\therefore$ Quadratic Equation is $4x^2 + \frac{11}{4}x + 1 = 0$

$\therefore$ The value of $(\alpha + \beta)^2 - 3\alpha\beta = \frac{121}{256} - \frac{3}{4} = -\frac{71}{256}$

**18.** Given $a + \alpha = 1$, $b + \beta = 2$ and $\alpha f(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ then value of $\dfrac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$

**Ans.** 2

**Sol.** $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ ........ (i)

$x \to \frac{1}{x}$

$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x$ ....... (ii)

(i) + (ii)

$(a + \alpha)\left[f(x)+f\left(\frac{1}{x}\right)\right] = \left(x+\frac{1}{x}\right)(b + \beta)$

$$\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}} = \frac{2}{1} = 2$$

**19.** Find the maximum value of 'k' for which the maximum value of variance of 10 elements is 10 in which 9 values are 1 and one value of is k. (Where k is integer)

**Ans.** 11

**Sol.** $\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$\sigma^2 = \frac{(9+k^2)}{10} - \left(\frac{9+k}{10}\right)^2 < 10$

$(90 + k^2)10 - (81 + k^2 + 8k) < 1000$

$90 + 10k^2 - k^2 - 18k - 81 < 1000$

$9k^2 - 18k + 9 < 1000$

$(k - 1)^2 < \frac{1000}{9} \Rightarrow k - 1 < \frac{10\sqrt{10}}{3}$

$k < \frac{10\sqrt{10}}{3} + 1$

Maximum integral value of k = 11

**20.** Distance of P(x, y) from (5,0) is thrice as distance of P(x, y) from (–5,0). If locus of P is circle with radius 'r' then final the value of $4r^2$.

**Ans.** 56.25

**Sol.** Internal point which divide (5,0) & (–5,0) in the ratio 3 : 1 is $\left(\frac{-5}{2},0\right)$ External point which divide (5,0) & (–5,0) in the ratio 3 : 1 is (–10,0)

$$2r = \left(\frac{-5}{2}+10\right) = \frac{15}{2} = 7.5$$

$$(2r)^2 = 56.25$$

**21.** Four numbers whose sum is $\frac{65}{12}$ are in G.P. Sum of their reciprocals is $\frac{65}{18}$ and product of first three of them is 1. If third term is $\alpha$ then find value of $2\alpha$.

**Ans.** 3

**Sol.** $a, ar, ar^2, ar^3$

$$a + ar + ar^2 + ar^3 = \frac{65}{12} \qquad \text{...... (i)}$$

$$\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}+\frac{1}{ar^3}=\frac{65}{18}$$

$$\frac{1}{a}\left(\frac{r^3+r^2+r+1}{r^3}\right) = \frac{65}{18} \qquad \text{....... (ii)}$$

$$\frac{(i)}{(ii)},\ a^2r^3 = \frac{18}{12} = \frac{3}{2}$$

$$a^3r^3 = 1 \Rightarrow a\left(\frac{3}{2}\right) = 1 \Rightarrow a = \frac{2}{3}$$

$$\frac{4}{9}r^3 = \frac{3}{2} \Rightarrow r^3 = \frac{3^3}{2^3} \Rightarrow r = \frac{3}{2}$$

$$\alpha = ar^2 = \frac{2}{3}\cdot\left(\frac{3}{2}\right)^2 = \frac{3}{2}$$

$$2\alpha = 3$$

**22.** There are 10 students $S_1, S_2$, ........ $S_{10}$. Find the number of ways to form 3 groups $G_1, G_2, G_3$ such that all groups has at least 1 member and group $G_3$ has almost 3 members

**Ans.** 26650

**Sol.**

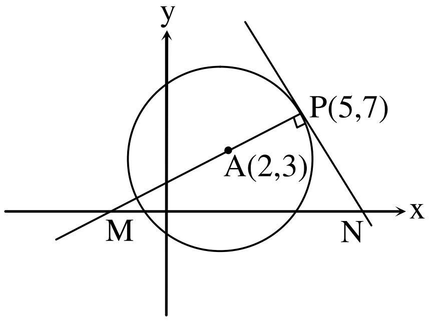
| A | B | C |
|---|---|---|
| 1 | 8 | 1 |
| 2 | 7 | 1 |
| ⋮ | ⋮ | ⋮ |
| 6 | 1 | 3 |

Ways to distribute in groups = ${}^{10}C_1({}^9C_1 + ....... {}^9C_8) + {}^{10}C_2({}^8C_1 + .....+ {}^8C_7)+ {}^{10}C_3({}^7C_1 + ..... + {}^7C_6)$
= 10(510) + 45(254) + 120(126)
= 26650

**23.** At point P(5, 7) on circle $(x - 2)^2 + (y - 3)^2 = 25$ a tangent and a normal is drawn. The area of triangle formed by this tangent normal with x axis is $\lambda$ then $24\lambda$ is

**Ans.** 1225

**Sol.**




equation of normal at P

$$(y - 7) = \left(\frac{7-3}{5-2}\right)(x - 5)$$

$$3y - 21 = 4x - 20$$

$$\Rightarrow 4x - 3y + 1 = 0 \qquad \text{........ (i)}$$

$$\Rightarrow M\left(-\frac{1}{4}, 0\right)$$

equation of tangent at P

$$(y - 7) = -\frac{3}{4}(x - 5)$$

$$4y - 28 = -3x + 15$$

$$\Rightarrow 3x + 4y = 43 \qquad \text{......... (ii)}$$

$$\Rightarrow N\left(\frac{43}{3}, 0\right)$$

hence $ar(\Delta PMN) = \frac{1}{2} \times MN \times 7$

$$l = \frac{1}{2} \times \frac{175}{12} \times 7$$

$$\Rightarrow 24\lambda = 1225$$